

On the Effect of Local Attraction in modifying the apparent form of the Earth, and some Remarks on the Real Existence of Diversity of Form, independent of their Effect. By Lieut. J. F. Tennant, Bengal Engineers, F.R.A.S., First Assistant G. T. Survey of India.

"In a paper I sent some time ago to the Society, I entered on an examination of the practical value of the results deduced by Mr. Pratt from his 'Estimate of Himmaleyan Attraction,' and showed that he failed in satisfying the geodesical data of the great longitudinal series.

"I have since received a letter from Mr. Pratt pointing out that I have extended his result beyond the limit he intended it to have. The ellipse given there would appear to be only a section of the supposed protuberant surface on that arc. It is useless for geodesical purposes, and I think there is no evidence that it is a real form at all.

"If, moreover, we suppose it to start at the southern extremity, cutting the mean ellipse, it will lie entirely within the mean surface, and can in any case only cut that surface at one point. The estimated projection of the surface at its centre above its chord is not then a measure of its difference from the general surface, as the chords of the two surfaces will be totally different. An ellipse representing a protuberance would generally require to have the extremity of its major axis out of the equatorial point, and this would satisfy three arcs. Hence the error into which I fell in interpreting Mr. Pratt's ellipse.

"The letter to which I have referred has again drawn my attention to the paper in the *Philosophical Transactions*; and in endeavouring to account for results so unexpectedly large, I have been led to the conclusion, that with Mr. Pratt's Hypothetical Attractions a further and very sensible correction to the measured lengths would be needed, and to some result on the modification of the mean figure, by attracting masses which will, I trust, be considered interesting.

"It is sufficiently shown in Mr. Pratt's paper that the order of error arising from neglecting the consideration that the visible side is projected on an erroneous plane is rejectaneous. Indeed it will appear evident that the neglect of $\frac{dR}{d\theta}$ in the expression

$$\frac{ds}{d\theta} = \sqrt{R^2 + \left(\frac{dR}{d\theta}\right)^2}$$

will be rejectaneous (in all cases where the attraction is not far more certainly known than from Mr. Pratt's formula,) in the following investigation.

"I purpose therefore assuming that the attraction of the Himmaleh may be sufficiently represented by a force varying inversely

as the chord of an arc drawn from the station perpendicular to the equivalent attracting prism (art. 50 of Mr. Pratt's paper) and, further, that it acts in the direction of the chord, which assumptions, while they have no sensible influence in modifying the attractions within the limits of the two arcs, are essential to the extension of the law farther from the prism.

"If, then, θ be the arc distance of any point from the prism, the deflection of the plumb-line would be

$$\frac{7235'' \cos \frac{1}{2} \theta}{2 R \sin \frac{1}{2} \theta} = \phi$$

or,

$$\phi = \frac{7235''}{2 R} \cdot \cot \frac{1}{2} \theta.$$

"Also $ds = R d\theta$.

"Therefore the extremity of ds would be elevated by

$$- \tan \phi ds = - \frac{7235 \tan 1''}{2 R} \cdot R \cot \frac{1}{2} \theta \cdot d\theta$$

above its true place were the attraction eliminated.

"Hence,

$$d \cdot \delta R = - 7235 \tan 1'' \cdot \cot \frac{1}{2} \theta \cdot \frac{d\theta}{2}$$

$$\delta R = 7235 \tan 1'' \int \cot \frac{1}{2} \theta \cdot d\theta$$

or if δR be in feet, R being miles,

$$\delta R = - \frac{38200800}{M} \tan 1'' \{ \log \sin \frac{1}{2} \theta + C \}$$

"If δR be supposed to vanish when $\theta = \frac{\pi}{2}$ we shall have

$$\begin{aligned} \delta R &= \frac{38200800}{M} \tan 1'' \{ \log \sin 45^\circ - \log \sin \frac{1}{2} \theta \} \\ &= \log^{-1} 2.6298630 \{ 9.8494850 - \log \sin \frac{1}{2} \theta \} \end{aligned}$$

"Now, taking $R = 3960$ miles, we should have at

$$\begin{aligned} \text{Damergida } \theta &= 13^\circ 55' \therefore \frac{1}{2} \theta = 6^\circ 57' 30'' \\ \text{Kalianpur } \theta &= 8^\circ 10' \therefore \frac{1}{2} \theta = 4^\circ 5' 0'' \\ \text{Kaliana } \theta &= 3^\circ 13' \therefore \frac{1}{2} \theta = 1^\circ 36' 30'' \end{aligned}$$

"Whence the radius vector would have been increased at

$$\left. \begin{aligned} \text{Damergida by } \delta R &= 326.7 \text{ feet} \\ \text{Kalianpur by } \delta R &= 425.4 \text{ ,,} \\ \text{Kaliana by } \delta R &= 597.6 \text{ ,,} \end{aligned} \right\} A$$

"Also, since

$$- \frac{38200800}{M} \tan 1'' \int_{\pi}^{\frac{\pi}{2}} \cot \frac{1}{2} \theta \frac{d\theta}{2} = 64.2 \text{ feet;}$$

this would be the increment of the radius vector from the antipodes of the centre of the prism to the great circle, whose pole is its centre, which is the origin of the quantities A .

"If these amounts A be the true increments of the radius, then the southern arc would require a correction of -35.3 feet, and the northern of -53.9 feet. Small as these are in comparison with the corrections necessary to make Mr. Pratt's attractions consistent with the general figure, they are far larger than those he has found in his paper.

"If we knew the true law of any attraction, we could assign ϕ as a function of λ and L , the geocentric latitude and longitude, and R_1 and s being measured to and on the apparent surface, we should have

$$d \cdot R_1 = \frac{dR}{d\lambda} \cdot d\lambda + \tan \phi \sqrt{\left(\frac{ds}{d\lambda}\right)^2 \cdot d\lambda^2 + \left(\frac{ds}{dL}\right)^2 \cdot dL^2}$$

"One result of this perturbation of the radius vector is, that every mass whose tendency is to raise the horizon towards itself, will cause a sensible protuberance of surface, and each which depresses the horizon will cause a depression. This disturbance of the surface is independent of its *real* form. The mere superposition of the mass of the Himmaleh, as estimated by Mr. Pratt without any alteration in the rest of the surface, would cause the surface at Kalia to appear depressed by nearly 600 feet.

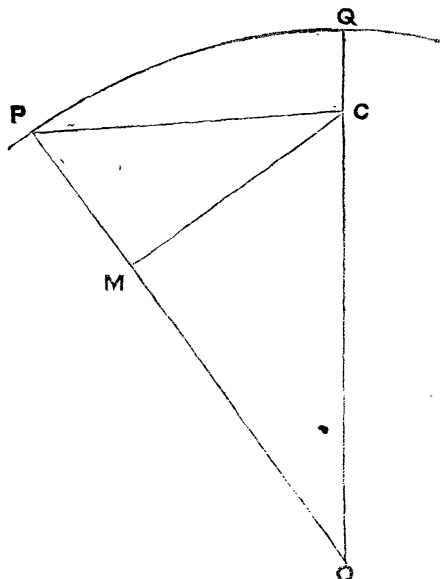
"It is evident that we have here a form of protuberance which, while entirely the result of attraction, is insensible to levelling operations entirely. It therefore differs entirely from the physical changes which are appealed to for the probability of real changes. Every mountain mass, by virtue of its attraction, causes a protuberance which is removed by the elimination of the effects of the attraction; so does every mass internally of greater density than the average of the stratum, but all these changes are dependent on the local disturbances of the plumb-line, and are not included in the tacit definition given by Mr. Pratt. The only change reconcilable with his process of eliminating the effect of irregular disturbance is a form so limited as that it shall have the power of keeping the vertical always normal to its surface. The only form, I believe, which possesses this property is an ellipsoidal shell concentric with the general form, and which would, therefore, merge in it. A protuberance of 157 feet at the centre of an arc, $11\frac{1}{2}$ degrees in length, and merging in the figure at both ends, would not have this effect any more than a flat rounded hill.

"If I am correct in this belief, it is a necessary corollary that every apparent local protuberance will be an effect of the neglect in part, or *in toto*, of the irregular deflections of the plumb-line, and that did our imperfect knowledge permit us entirely to eliminate this source of error, we should have a figure of equilibrium as the result independent of any speculations as to the processes by which the present distribution of matter in the earth has been attained.

"I will conclude with an examination of the mass which, acting in the centre of an arc, would produce an elevation similar to that which is the asserted result of Mr. Pratt's investigations.

"I shall in this consider the surface as sufficiently satisfying the equation $ds = R d\theta$, where R is constant, and also that the mass causing deflection can be considered as concentrated in a point. The former hypothesis can cause no sensible error, and the latter will answer my purpose, which is (a rigorous investigation being out of the question) merely to show how small a mass will suffice to produce the result.

"In the accompanying figure let PQ represent the curved surface; O its centre; $OP = OQ = R$ the radius. Let C be the centre of the attracting mass, and $CQ = r$. Let the angle $POC = \theta$, the deflection at $P = \phi$, and the deflecting mass which I will call the mass $C = \text{earth's mass} \times \frac{1}{m}$.



"The attraction of C on P : attraction of the earth on $P :: OP^2 : m PC^2$.

"Consequently the resolved portion in the direction of the tangent at $P =$ attraction of earth on

$$P \cdot \frac{OP^2}{m PC^2} \times \frac{CM}{CP}$$

CM being perpendicular to PO ; and the portion parallel to PO is rejectaneous compared with the earth's attraction.

"Now,

$$CM = CO \sin \theta = (R - r) \sin \theta$$

and

$$\begin{aligned} PC^2 &= (R - r)^2 + R^2 - 2R(R - r) \cos \theta \\ &= r^2 + 2R(R - r) \sin^2 \frac{1}{2} \theta \end{aligned}$$

therefore

$$\tan \phi = \frac{R^2 (R - r) \sin \theta}{m \{ r^2 + 2R(R - r) \sin^2 \frac{1}{2} \theta \}^{\frac{3}{2}}} = \frac{2R^2 (R - r) \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta}{m \{ r^2 + 2R(R - r) \sin^2 \frac{1}{2} \theta \}^{\frac{3}{2}}}$$

"Hence,

$$d\delta R - R d\theta \tan \phi = - \frac{2R^3 (R - r) \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta}{m \{ r^2 + 2R(R - r) \sin^2 \frac{1}{2} \theta \}^{\frac{3}{2}}}$$

"Integrating, we find the value of

$$\begin{aligned} \delta R \text{ at } Q &= \Delta R = \frac{2R^2}{m} \int_0^\theta \frac{2R(R - r) \sin \frac{1}{2} \theta \cos \frac{1}{2} \theta \cdot d\theta}{\{ r^2 + 2R(R - r) \sin^2 \frac{1}{2} \theta \}^{\frac{3}{2}}} \\ &= \frac{R^2}{m} \left\{ \frac{1}{r} - \frac{1}{\sqrt{2R(R - r) \sin^2 \frac{1}{2} \theta + r^2}} \right\} \end{aligned}$$

C

or if ΔR be feet,

$$\Delta R = \frac{5280 R^2}{m} \left\{ \frac{1}{r} - \frac{1}{\sqrt{2 R (R-r) \sin^2 \frac{1}{2} \theta + r^2}} \right\}$$

“ Let

$$r = 52.8 \text{ feet} = 0.01 \text{ mile}$$

$$R = 3960 \text{ miles}$$

$$\Delta R = 157 \text{ feet.}$$

2θ = difference of latitudes of the extremities

$$= 11^\circ 27' 32'' \text{ or } \frac{1}{2} \theta = 2^\circ 51' 53''$$

“ Then

$$\begin{aligned} \Delta R &= \frac{5280 R^2}{m} \{ 100 - 0.0035728 \} \\ &= \frac{\log^{-1} 12.9180093}{m} = 157 \end{aligned}$$

therefore

$$\begin{aligned} m &= \frac{\log^{-1} 12.9180093}{157} \\ &= \log^{-1} 10.7221096 \end{aligned}$$

“ If we suppose this mass half the mean density of the earth, the sphere containing the necessary volume would be 1.351 miles in radius.

“ Such a protuberance then might, in the case of the Indian arc, easily be produced by the elevated masses of Central India, whose greater volume would be counteracted by their diffusion of a large space.

“ The attraction, then, of this mass on a station situated like Kalianpur, where $\theta = 20' 9''$ would be

$$\phi = \tan^{-1} \frac{R^2 (R-r) \sin 20' 9''}{m \{ r^2 + 2 R (R-r) \sin^2 10' 4'' . 5 \}^{\frac{3}{2}}} = 6'' . 67$$

“ This, in conjunction with the correction to the measured arc, would satisfy (even with Mr. Pratt's attractions) very closely the southern arc (Damergida to Kalianpur), considered as measured on the average spheroid. That, however, is accidental. Mr. Pratt's form does not, like this, meet the general ellipse in the ends of the arc.

“ Since, then, we can account for apparent deviations of the surface of the earth from the mean form by, as I think I have shown, the effects of the attraction of small masses of matter; and we have, and can have, no evidence of the real existence of deviations which are independent of this source and real; it appears to me not consistent with the principles of induction to assign such as we may imagine we find to the latter cause.

“ I have shown in my former paper that the ellipse given by Mr. Pratt for the Indian meridian is useless for geodesical purposes. I have now, I think, shown reason to believe that there is

no evidence which will warrant our considering that any real departure from the mean form exists.

“The following conclusion is not without its importance: It is impossible to place any dependence on a figure deduced from neighbouring arcs, even within their own limits, as considerable deviations may be caused by the attraction of small masses favourably situated; and the only figure of any geodesical or astronomical importance is that determined as usual from the consideration of a considerable number of arcs situated in various circumstances as regards sources of probable local disturbances.”

Extract of a Letter from M. Schwabe to Mr. Carrington.

(Translated freely.)

“In the address of the President, contained in your *Monthly Notices*, the question is inadvertently asked, of what led me originally to undertake observations of the sun. It was in consequence of a representation of my old and valued friend Harding of Göttingen, who wrote to me that there was a great want of physical observations of the sun, that the subject presented an almost unworked field, and that labour therein might be rewarded by the discovery of a planet interior to *Mercury*. The communications of Harding and of a friend in Berlin led me also to inquire into the rotation of the sun, in which I found some difficulty from the instability of the spots. Nevertheless, I convinced myself that the opinion of Schroeter, that the spots had an independent motion, was unfounded, and that the apparent movement was attributable rather to the enlarging of one side and the filling in of the other side of a nuclear spot, whereby the estimated centre was shifted. My enumerations, however, early led me to notice a tolerably regular increase and decrease in the number of the spots, so that I may compare myself to Saul, who went out to seek his father's asses, and found a kingdom.”

Report of the Syndicate appointed to visit the Observatory of Cambridge (May 30, 1857).

We extract the following passages from this Report:—

“The transit observations in the year 1856 amounted to 1865, and the circle observations to 1121.

“The total number of comparisons in R.A. (with the Northumberland equatoreal), is 695, and of those in N.P.D. 585.